

A note on initial state entanglement in inflationary cosmology

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Abstract

We give a new interpretation of the effect of initial state entanglement on the spectrum of vacuum fluctuations. We consider an initially entangled state between two free massive scalar fields in de Sitter space. We construct the initial state by making use of a Bogoliubov transformation between the Bunch-Davies vacuum and a four mode squeezed state, and then derive the exact power spectrum for one of the scalar fields. We demonstrate that an oscillatory spectrum hardly appears for the initially entangled state unless an ad hoc absolute value of the Bogoliubov coefficients is chosen.

Contents

1	Introduction	1
2	Four mode squeezed state in de Sitter space	3
2.1	The set-up	3
2.2	The Bogoliubov transformation	4
2.3	The spectrum in the Heisenberg picture	5
2.4	The spectrum in the Schrödinger picture	5
3	Summary and discussion	7

1 Introduction

Historically, quantum entanglement has been one of the most fascinating features predicted by quantum mechanics since Einstein-Podolsky-Rosen (EPR) pointed out that performing a local measurement may affect the outcome of local measurements instantaneously beyond the light cone in 1935 [1]. After a convincing test that the quantum entanglement is a fundamental aspect of quantum mechanics by measuring correlations of linear polarizations of pairs of photons [2, 3] was performed by Aspect et al in 1981, more attention has been paid to how to make use of quantum entanglement of EPR pairs in quantum cryptography and quantum teleportation (see [4] and references therein).

Entanglement entropy has now been established as a suitable measure of the degree of entanglement of a quantum system. It has become a useful tool in quantum field theory to characterize the nature of long range correlations and been developed in condensed matter physics, quantum information and high energy physics. Recently, to discuss a gravity dual of quantum entanglement in de Sitter space, Maldacena and Pimentel developed an explicit method to calculate the entanglement entropy in a quantum field theory in the Bunch-Davies vacuum of the de Sitter space [5]. They showed that quantum entanglement can exist between two causally disconnected open charts in de Sitter space. Since the interior of a nucleated bubble can be described by the open chart [6], their result infers that the regions inside and outside of the bubble are entangled. If we regard the two open charts as two separated bubbles, the two bubbles are entangled. Then, the spectrum of vacuum fluctuations in one of the open charts was examined in [7] and it was found that the quantum entanglement affects the shape of the spectrum on large scales comparable to or greater than the curvature radius, whereas the spectrum on small scales remains intact [7, 8]. This is because the Bogoliubov transformation between the Bunch-Davies vacuum and the vacua of the open charts depends

on the wavenumber and vanishes in the small scale limit. Thus there is no entanglement on small scales. This result is consistent with the fact that there should be no difference between the open chart vacuum and the Bunch-Davies vacuum, since spacetime looks flat in the small scale limit. In other words, this result that there is no entanglement on small scales is reasonable as long as bubbles are nucleated in vacuum, with no particle production on small scales.

The research of [9], however, discussed that oscillatory spectra may appear as the effect of quantum entanglement even on small scales if we consider an initially entangled state between two free massive scalar fields in de Sitter space. If the amplitude of oscillations is small enough, the spectrum cannot be distinguished from a flat spectrum. On the other hand, the research of [10] studied two de Sitter spaces instead of the two scalar fields and showed that an initial state entanglement between two causally disconnected de Sitter spaces may remain on small scales. In the bubble universes or so called multiverse, an infinite number of universes can be produced once inflation happens [11, 12, 13, 14, 15, 16]. In such a situation, some bubbles may be nucleated as entangled pairs of bubbles and their initial states may be entangled with their partner bubbles¹. Then, some effects of the entanglement with unobservable bubbles may appear in the CMB of our bubble. In fact, the research of [22] showed that the quantum entanglement may be imprinted on the cosmological observables.

Since the initially entangled state considered in [22] corresponds to truncation of a squeezed state of pairs of n -particles, the degree of entanglement is represented by the Bogoliubov coefficients between the Bunch-Davies vacuum and the squeezed state. Thus one may expect to obtain oscillatory spectra with an appropriate choice of the Bogoliubov coefficients β as demonstrated in [9], although such oscillatory spectra are not solely due to the entanglement because initially non-entangled states also produce the oscillatory spectra because of the Bogoliubov coefficients β [22]. The research of [22] showed that the relative phase between the coefficients α and β due to quantum interference may distinguish the spectrum of initially entangled states from the one of non-entangled.

The computation in [9] is done in the Schrödinger formalism, so it is not easy to see the physical origin of the oscillatory behavior in the spectrum. In this paper, we instead adopt the Heisenberg formalism and clarify what causes the oscillatory behavior in the spectrum when the initial state is entangled. We then give a new interpretation of the result.

¹The nucleation of bubbles is analogous to Schwinger pair production [17, 18, 19, 20, 21].

2 Four mode squeezed state in de Sitter space

In [9], an initially four mode squeezed state of two free massive scalar fields in de Sitter space was discussed in the Schrödinger formalism. As a result, oscillatory spectra appeared. As we show below, this state describes an entangled state between the two scalar fields. Here, we discuss their system in the Heisenberg formalism and try to figure out what caused the oscillatory behavior in the spectrum.

2.1 The set-up

We consider two free massive scalar fields $\phi_{\mathbf{k}}$ and $\chi_{\mathbf{k}}$ in de Sitter with the action [9]

$$S = \frac{1}{2} \int d\eta \sum_{\mathbf{k}} \left[a^2 (\phi'_{\mathbf{k}} \phi_{\mathbf{k}}^* - k^2 \phi_{\mathbf{k}} \phi_{\mathbf{k}}^*) - a^4 m_\phi^2 \phi_{\mathbf{k}} \phi_{\mathbf{k}}^* + a^2 (\chi'_{\mathbf{k}} \chi_{\mathbf{k}}^* - k^2 \chi_{\mathbf{k}} \chi_{\mathbf{k}}^*) - a^4 m_\chi^2 \chi_{\mathbf{k}} \chi_{\mathbf{k}}^* \right] \quad (2.1)$$

where the scale factor $a = -1/(H\eta)$ and a prime denotes derivative with respect to the conformal time η . \mathbf{k} is the wave vector. The canonical conjugate momentum for $\phi_{\mathbf{k}}$ and $\chi_{\mathbf{k}}$ is give by

$$P_{\phi \mathbf{k}} = \frac{\partial S}{\partial \phi'_{\mathbf{k}}} = a^2 \phi_{\mathbf{k}}^*, \quad P_{\chi \mathbf{k}} = \frac{\partial S}{\partial \chi'_{\mathbf{k}}} = a^2 \chi_{\mathbf{k}}^*. \quad (2.2)$$

Each field is expanded as

$$\begin{aligned} \phi_{\mathbf{k}} &= a_{\mathbf{k}} u_k(\eta) + a_{-\mathbf{k}}^\dagger u_k^*(\eta), & [a_{\mathbf{k}}, a_{\mathbf{p}}^\dagger] &= \delta_{\mathbf{k}, \mathbf{p}}, \\ \chi_{\mathbf{k}} &= b_{\mathbf{k}} v_k(\eta) + b_{-\mathbf{k}}^\dagger v_k^*(\eta), & [b_{\mathbf{k}}, b_{\mathbf{p}}^\dagger] &= \delta_{\mathbf{k}, \mathbf{p}}, \end{aligned} \quad (2.3)$$

where k is the magnitude of the wave vector \mathbf{k} . The different masses of m_ϕ and m_χ produce different mode functions u_k and v_k , which satisfy

$$\begin{aligned} u_k'' + \left(k^2 + m_\phi^2 a^2 - \frac{a''}{a} \right) u_k &= 0, & u_k u_k^{*'} - u_k^* u_k' &= \frac{i}{a^2}, \\ v_k'' + \left(k^2 + m_\chi^2 a^2 - \frac{a''}{a} \right) v_k &= 0, & v_k v_k^{*'} - v_k^* v_k' &= \frac{i}{a^2}. \end{aligned} \quad (2.4)$$

where we used the canonical conjugate relations $[\phi_{\mathbf{k}}, P_{\phi \mathbf{p}}] = i\delta_{\mathbf{k}, \mathbf{p}}$, $[\chi_{\mathbf{k}}, P_{\chi \mathbf{p}}] = i\delta_{\mathbf{k}, \mathbf{p}}$ for the Klein-Gordon normalization respectively. The mode functions that realize the Bunch-Davies vacuum at $\eta \rightarrow -\infty$ are respectively given by

$$\begin{aligned} u_k &= \frac{\sqrt{\pi}}{2} \sqrt{-\eta} H_{\nu_\phi}^{(1)}(-k\eta), & \nu_\phi &= \sqrt{\frac{9}{4} - \frac{m_\phi^2}{H^2}}, \\ v_k &= \frac{\sqrt{\pi}}{2} \sqrt{-\eta} H_{\nu_\chi}^{(1)}(-k\eta), & \nu_\chi &= \sqrt{\frac{9}{4} - \frac{m_\chi^2}{H^2}}. \end{aligned} \quad (2.5)$$

Note that we assumed $m_\phi^2, m_\chi^2 < 9/4H^2$ for simplicity. Then the canonical conjugate momentum Eq. (2.2) is expressed as

$$P_{\phi\mathbf{k}} = a^2 \left(a_{-\mathbf{k}} u'_k + a_{\mathbf{k}}^\dagger u_k^* \right), \quad P_{\chi\mathbf{k}} = a^2 \left(b_{-\mathbf{k}} v'_k + b_{\mathbf{k}}^\dagger v_k^* \right). \quad (2.6)$$

By using Eqs. (2.3) and (2.6), the annihilation and creation operators are written as

$$\begin{aligned} a_{\mathbf{k}} &= u_k^* \frac{\partial}{\partial \phi_{-\mathbf{k}}} - i a^2 u_k^* \phi_{\mathbf{k}}, & a_{-\mathbf{k}}^\dagger &= -u_k \frac{\partial}{\partial \phi_{-\mathbf{k}}} + i a^2 u'_k \phi_{\mathbf{k}}, \\ b_{\mathbf{k}} &= v_k^* \frac{\partial}{\partial \chi_{-\mathbf{k}}} - i a^2 v_k^* \chi_{\mathbf{k}}, & b_{-\mathbf{k}}^\dagger &= -v_k \frac{\partial}{\partial \chi_{-\mathbf{k}}} + i a^2 v'_k \chi_{\mathbf{k}}. \end{aligned} \quad (2.7)$$

The vacuum state is the state annihilated by both a_k and b_k . So if we denote the vacuum for $\phi_{\mathbf{k}}$ by $|0\rangle_\phi$ and for $\chi_{\mathbf{k}}$ by $|0\rangle_\chi$, the vacuum for the total system is

$$|0\rangle = |0\rangle_\phi |0\rangle_\chi. \quad (2.8)$$

2.2 The Bogoliubov transformation

Let us consider a state $|\psi\rangle$ defined by Bogoliubov transformations that mix the operators $a_{\mathbf{k}}$ with $b_{\mathbf{k}}$

$$\tilde{a}_{\mathbf{k}} = \alpha_k a_{\mathbf{k}} + \beta_k b_{-\mathbf{k}}^\dagger, \quad \tilde{b}_{\mathbf{k}} = \alpha_k b_{\mathbf{k}} + \beta_k a_{-\mathbf{k}}^\dagger, \quad (2.9)$$

that is

$$\tilde{a}_{\mathbf{k}} |\psi\rangle = \tilde{b}_{\mathbf{k}} |\psi\rangle = 0, \quad (2.10)$$

where

$$|\alpha_k|^2 - |\beta_k|^2 = 1. \quad (2.11)$$

This state $|\psi\rangle$ is then written by

$$|\psi\rangle = N \exp \left[- \sum_{\mathbf{k}} \frac{\beta_k}{\alpha_k} a_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger \right] |0\rangle, \quad (2.12)$$

where N is the normalization factor. This describes a squeezed state of pairs of n -particles between the two scalar fields $\phi_{\mathbf{k}}$ and $\chi_{\mathbf{k}}$. If we expand the exponent in $|\psi\rangle$, we have

$$\begin{aligned} |\psi\rangle &= N \left(1 - \frac{\beta_k}{\alpha_k} a_{\mathbf{k}}^\dagger b_{-\mathbf{k}}^\dagger + \cdots \right) |0\rangle \\ &= N \left(|0\rangle_\phi |0\rangle_\chi - \frac{\beta_k}{\alpha_k} |1\rangle_\phi |1\rangle_\chi + \frac{1}{2} \left(\frac{\beta_k}{\alpha_k} \right)^2 |2\rangle_\phi |2\rangle_\chi + \cdots \right). \end{aligned} \quad (2.13)$$

This is an entangled state of the $\mathcal{H}_\phi \otimes \mathcal{H}_\chi$ space. If we truncate the above at the one particle state order, we get a standard description of composite system of an entangled state between the vacuum and one-particle states. Thus the degree of entanglement corresponds to the Bogoliubov coefficients and it may depend on the wavenumber k .

2.3 The spectrum in the Heisenberg picture

Now, we calculate the spectrum of one of the scalar fields $\phi_{\mathbf{k}}$ in the state $|\psi\rangle$ in Eq. (2.12). Eq. (2.3) and its complex conjugate are expanded in terms of $\tilde{a}_{\mathbf{k}}$ and $\tilde{b}_{\mathbf{k}}$ as

$$\phi_{\mathbf{k}} = \left(\alpha_k^* \tilde{a}_{\mathbf{k}} - \beta_k \tilde{b}_{-\mathbf{k}}^\dagger \right) u_k + \left(\alpha_k \tilde{a}_{-\mathbf{k}}^\dagger - \beta_k^* \tilde{b}_{\mathbf{k}} \right) u_k^*, \quad (2.14)$$

$$\phi_{\mathbf{k}}^\dagger = \left(\alpha_k \tilde{a}_{\mathbf{k}}^\dagger - \beta_k^* \tilde{b}_{-\mathbf{k}} \right) u_k^* + \left(\alpha_k^* \tilde{a}_{-\mathbf{k}} - \beta_k \tilde{b}_{\mathbf{k}}^\dagger \right) u_k. \quad (2.15)$$

Then the spectrum of vacuum fluctuations is calculated as

$$\langle \psi | \phi_{\mathbf{k}} \phi_{\mathbf{k}}^\dagger | \psi \rangle = |u_k|^2 (|\alpha_k|^2 + |\beta_k|^2) = |u_k|^2 (1 + 2|\beta_k|^2), \quad (2.16)$$

where we used Eq. (2.10). This is the exact spectrum for the field $\phi_{\mathbf{k}}$. Interestingly, we find that the mode function v_k does not come in the spectrum as far as we focus on the spectrum for $\phi_{\mathbf{k}}$. Thus, the possible origin of oscillatory spectra has to be in the absolute value of the Bogoliubov coefficients $|\beta_k|$.

2.4 The spectrum in the Schrödinger picture

Let us assume that $|\psi\rangle$ in the Schrödinger picture is expressed of the form [9]

$$\psi = N \exp \left[-\frac{1}{2} \sum_{\mathbf{k}} A_k \phi_{\mathbf{k}} \phi_{-\mathbf{k}} - \frac{1}{2} \sum_{\mathbf{k}} B_k \chi_{\mathbf{k}} \chi_{-\mathbf{k}} - \frac{1}{2} \sum_{\mathbf{k}} C_k (\phi_{\mathbf{k}} \chi_{-\mathbf{k}} + \chi_{\mathbf{k}} \phi_{-\mathbf{k}}) \right]. \quad (2.17)$$

The last term proportional to C_k corresponds to an initial state entanglement between $\phi_{\mathbf{k}}$ and $\chi_{\mathbf{k}}$. Here we note that $\phi_{-\mathbf{k}} = \phi_{\mathbf{k}}^\dagger$ in the Heisenberg picture. So in the Schrödinger picture or in the coordinate representation, $\phi_{-\mathbf{k}} = \phi_{\mathbf{k}}^*$.

Now we try to check if the state ψ in the Schrödinger picture (2.17) corresponds to $|\psi\rangle$ in the Heisenberg picture (2.12). Plugging Eq. (2.7) into Eq. (2.9), we find

$$\begin{aligned} \tilde{a}_{\mathbf{k}} &= \alpha_k \left(u_k^* \frac{\partial}{\partial \phi_{-\mathbf{k}}} - i a^2 u_k^{*'} \phi_{\mathbf{k}} \right) + \beta_k \left(-v_k \frac{\partial}{\partial \chi_{-\mathbf{k}}} + i a^2 v_k' \chi_{\mathbf{k}} \right), \\ \tilde{b}_{\mathbf{k}} &= \alpha_k \left(v_k^* \frac{\partial}{\partial \chi_{-\mathbf{k}}} - i a^2 v_k^{*'} \chi_{\mathbf{k}} \right) + \beta_k \left(-u_k \frac{\partial}{\partial \phi_{-\mathbf{k}}} + i a^2 u_k' \phi_{\mathbf{k}} \right). \end{aligned} \quad (2.18)$$

If we operate Eq. (2.18) on Eq. (2.17) and use Eq. (2.10), we get a system of four homogeneous equations

$$A_k u_k^* - \frac{\beta_k}{\alpha_k} C_k v_k = -i a^2 u_k^{*'}, \quad B_k v_k - \frac{\alpha_k}{\beta_k} C_k u_k^* = -i a^2 v_k', \quad (2.19)$$

$$B_k v_k^* - \frac{\beta_k}{\alpha_k} C_k u_k = -i a^2 v_k^{*'}, \quad A_k u_k - \frac{\alpha_k}{\beta_k} C_k v_k^* = -i a^2 u_k'. \quad (2.20)$$

Note that one of them is redundant. Those equations determine coefficients A_p , B_p and C_p in Eq. (2.17) as

$$A_k = \frac{1}{|u_k|^2} \frac{\beta_k^2 u_k v_k}{\alpha_k^2 u_k^* v_k^* - \beta_k^2 u_k v_k} - i a^2 \frac{u_k^{*'}}{u_k^*}, \quad (2.21)$$

$$B_k = \frac{1}{|v_k|^2} \frac{\beta_k^2 u_k v_k}{\alpha_k^2 u_k^* v_k^* - \beta_k^2 u_k v_k} - i a^2 \frac{v_k^{*'}}{v_k^*}, \quad (2.22)$$

$$C_k = \frac{\alpha_k \beta_k}{\alpha_k^2 u_k^* v_k^* - \beta_k^2 u_k v_k}, \quad (2.23)$$

where the relation of the Klein-Gordon normalization in (2.4) is used. Thus we find that both expressions in the Heisenberg and Schrödinger pictures Eqs. (2.12) and (2.17) completely agree with each other when the above (2.21), (2.22) and (2.23) are satisfied.

As we see Eqs. (2.21)~(2.23), those expressions look very complicated. In [9], they tried to derive Eqs. (2.21)~(2.23) approximately, and then calculated the power spectrum in the Schrödinger picture. So we try to check if the power spectrum agrees with Eq. (2.16). In the Schrödinger picture, the spectrum becomes [9]

$$\langle \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \rangle = \frac{(\text{Re} B_k)}{2\{(\text{Re} A_k)(\text{Re} B_k) - (\text{Re} C_k)^2\}}, \quad (2.24)$$

where $\text{Re} A_k$ is the real part of A_k and similarly for B_k and C_k .

Now we express each variable in A_k , B_k and C_k in the form²

$$\beta_k = b_k + i d_k, \quad u_k = x_k + i y_k, \quad v_k = z_k + i w_k, \quad (2.25)$$

where b_k, d_k, x_k, y_k, z_k and w_k are real. We plug the above into Eqs. (2.21), (2.22) and (2.23), and calculate Eq. (2.24). If we expand it in β_k up to the second order, we find that the Eq. (2.24) is written by

$$\frac{1}{2a^2} \frac{x^2 + y^2}{x'y - y'x} \left[1 - \frac{1}{a^2} \frac{1}{x'y - y'x} \frac{AX + BY}{X^2 + Y^2} + \frac{\alpha^2}{a^4} \frac{(x^2 + y^2)(z^2 + w^2)}{(x'y - y'x)(z'w - w'z)} \left(\frac{bX + dY}{X^2 + Y^2} \right)^2 \right] \quad (2.26)$$

where we omitted the index k for simplicity and defined

$$\begin{aligned} X &= \alpha^2 (xz - yw), & A &= (b^2 - d^2) (xz - yw) - 2bd (yz + xw), \\ Y &= -\alpha^2 (yz + xw), & B &= 2bd (xz - yw) + (b^2 - d^2) (yz + xw). \end{aligned} \quad (2.27)$$

If we use the Wronskian $yx' - xy' = 1/(2a^2)$, and rewrite the above in terms of $\alpha_k, \beta_k, u_k, v_k$ such as $X^2 + Y^2 = \alpha^4 |u_k|^2 |v_k|^2$, the spectrum (2.24) is then expressed as

$$\langle \phi_{\mathbf{k}} \phi_{-\mathbf{k}} \rangle = |u_k|^2 (1 + 2|\beta_k|^2 + \mathcal{O}(\beta_k^4)). \quad (2.28)$$

²Without loss of generality, we assume α_k is real.

We see that the expansion up to the second order of β_k completely agrees with the exact spectrum in Eq. (2.16). Thus, we find that the degree of freedom of an oscillation in the spectrum appeared in Eq. (2.24) is encoded in the absolute value of the Bogoliubov coefficients $|\beta_k|$ and then the oscillation can appear with an appropriate choice of the $|\beta_k|$. In fact, in [9], they choose the phase of ρ_k to be zero and which corresponds to a particular (rather ad hoc) choice of $|\beta_k|$.

3 Summary and discussion

We studied the spectrum of vacuum fluctuations of an initial state where two scalar fields are entangled by following the research [9], in which it was argued that oscillatory spectra would appear due to the entanglement. They considered an initially four mode squeezed state of the two free massive scalar fields $\phi_{\mathbf{k}}$, $\chi_{\mathbf{k}}$, one of them is assumed to be an inflaton field, in de Sitter space in the Schrödinger formalism. Since in their Schrödinger formalism it is not easy to see the physical origin of the oscillatory behavior in the spectrum, we instead adopted the Heisenberg formalism. In the Heisenberg formalism, the state considered in [9] is described by a Bogoliubov transformation that mixes a positive frequency function of $\phi_{\mathbf{k}}$ with a negative frequency function of $\chi_{\mathbf{k}}$. We then calculated the exact power spectrum for the ϕ field and clarified that the origin of an oscillatory spectrum has to be in the absolute value of the Bogoliubov coefficients $|\beta_k|$.

To confirm that our state in the Heisenberg picture is the same as the state studied in [9], we derived the wavefunction for our state in the Schrödinger picture in the same form as assumed in [9]. We note that the resultant wavefunction we obtained is exact and general for the class of entanglement considered in [9]. To show explicitly the identity between an expression for the spectrum given in terms of functions appearing in the wavefunction as given in Eq. (2.24) and the exact expression we obtained, we evaluated the former approximately by expanding it in terms of the Bogoliubov coefficients β_k . We found that the spectrum expanded up to the second order of the Bogoliubov coefficients β_k completely agrees with the exact spectrum we derived in the Heisenberg formalism. From those analyses, we conclude that oscillatory spectra due to the initial state entanglement can be produced only when the absolute value of the Bogoliubov coefficient $|\beta_k|$ is chosen to have an oscillatory feature in k .

We here stress that, on the contrary, an initially non-entangled state may naturally produce an oscillatory spectrum due to quantum interference if the initial state deviates from the Bunch-Davies vacuum as shown in [22]. For example, if we consider a squeezed state of $\phi_{\mathbf{k}}$ given by a Bogoliubov transformation of the Bunch-Davies vacuum, an oscillatory spectrum appears if the relative phase between the coefficients α_k and β_k depends on k even if there

is no oscillation in the absolute value of β_k . Thus we should not be confused the oscillatory spectra as an effect of entanglement. In order to see solely the effect of entanglement in the spectrum, we should rather see the effect of quantum interference that may distinguish the spectrum of initially entangled state from the one of non-entangled state [22].

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References

- [1] A. Einstein, B. Podolsky and N. Rosen, Phys. Rev. **47**, 777 (1935).
- [2] A. Aspect, P. Grangier and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).
- [3] A. Aspect, J. Dalibard and G. Roger, Phys. Rev. Lett. **49**, 1804 (1982).
- [4] R. Horodecki, P. Horodecki, M. Horodecki and K. Horodecki, Rev. Mod. Phys. **81**, 865 (2009) [quant-ph/0702225].
- [5] J. Maldacena and G. L. Pimentel, JHEP **1302**, 038 (2013) [arXiv:1210.7244 [hep-th]].
- [6] S. R. Coleman and F. De Luccia, Phys. Rev. D **21**, 3305 (1980).
- [7] S. Kanno, JCAP **1407**, 029 (2014) [arXiv:1405.7793 [hep-th]].
- [8] F. V. Dimitrakopoulos, L. Kabir, B. Mosk, M. Parikh and J. P. van der Schaar, JHEP **1506**, 095 (2015) [arXiv:1502.00113 [hep-th]].
- [9] A. Albrecht, N. Bolis and R. Holman, JHEP **1411**, 093 (2014) [arXiv:1408.6859 [hep-th]].
- [10] S. Kanno, J. P. Shock and J. Soda, JCAP **1503**, no. 03, 015 (2015) [arXiv:1412.2838 [hep-th]].
- [11] K. Sato, H. Kodama, M. Sasaki and K. i. Maeda, Phys. Lett. B **108**, 103 (1982).
- [12] A. Vilenkin, Phys. Rev. D **27**, 2848 (1983).
- [13] A. D. Linde, Mod. Phys. Lett. A **1**, 81 (1986).

- [14] A. D. Linde, Phys. Lett. B **175**, 395 (1986).
- [15] R. Bousso and J. Polchinski, JHEP **0006**, 006 (2000) [hep-th/0004134].
- [16] L. Susskind, In *Carr, Bernard (ed.): Universe or multiverse?* 247-266 [hep-th/0302219].
- [17] J. S. Schwinger, Phys. Rev. **82**, 664 (1951).
- [18] J. D. Brown and C. Teitelboim, Nucl. Phys. B **297** (1988) 787.
- [19] J. Garriga, S. Kanno, M. Sasaki, J. Soda and A. Vilenkin, JCAP **1212**, 006 (2012) [arXiv:1208.1335 [hep-th]].
- [20] J. Garriga, S. Kanno and T. Tanaka, JCAP **1306**, 034 (2013) [arXiv:1304.6681 [hep-th]].
- [21] M. B. Fröb, J. Garriga, S. Kanno, M. Sasaki, J. Soda, T. Tanaka and A. Vilenkin, JCAP **1404**, 009 (2014) [arXiv:1401.4137 [hep-th]].
- [22] S. Kanno, arXiv:1506.07808 [hep-th].